Exercise 1E: Solutions

1 Let k be the number of kilometres travelled in a day. The unlimited kilometre alternative will become more attractive when $0.32k + 63 > 108$.

Solve for $0.32k + 63 = 108$:

$$\begin{aligned} 0.32k &= 108 - 63 \\ &= 45 \\ k &= \frac{45}{0.32} = 140.625 \end{aligned}$$

The unlimited kilometre alternative will become more attractive when you travel more than 140.625 km.

2 Let g be the number of guests. Solve for the equality.

$$\begin{aligned} 300 + 43g &= 450 + 40g \\ 43g - 40g &= 450 - 300 \\ 3g &= 150 \\ g &= 50 \end{aligned}$$

Company A is cheaper when there are more than 50 guests.

3 Let a be the number of adults and c the number of children.

$$\begin{aligned} 45a + 15c &= 525\ 000 \\ a + c &= 15\ 000 \end{aligned}$$

Multiply the second equation by 15.

$$\begin{aligned} 45a + 15c &= 525\ 000 \quad (1) \\ 15a + 15c &= 225\ 000 \quad (2) \end{aligned}$$

$(1) - (2)$:

$$\begin{aligned} 30a &= 300\ 000 \\ a &= 10\ 000 \end{aligned}$$

$10\ 000$ adults and 5000 children bought tickets.

4 Let $\$m$ be the amount the contractor paid a man and $\$b$ the amount he paid a boy.

$$\begin{aligned} 8m + 3b &= 2240 \\ 6m + 18b &= 4200 \end{aligned}$$

Multiply the first equation by 6.

$$\begin{aligned} 48m + 18b &= 13\ 440 \quad (1) \\ 6m + 18b &= 4200 \quad (2) \end{aligned}$$

$(1) - (2)$:

$$\begin{aligned} 42m &= 9240 \\ m &= 220 \end{aligned}$$

Substitute into the first equation:

$$\begin{aligned} 8 \times 220 + 3b &= 2240 \\ 1760 + 3b &= 2240 \\ 3b &= 2240 - 1760 \\ &= 480 \\ b &= 160 \end{aligned}$$

He paid the men $\$220$ each and the boys $\$160$.

5 Let the numbers be x and y.

$$\begin{aligned} x + y &= 212 \quad (1) \\ x - y &= 42 \quad (2) \end{aligned}$$

$(1) + (2)$:

$$\begin{aligned} 2x &= 254 \\ x &= 127 \end{aligned}$$

$$127 + y = 212$$

$y = 85$

The numbers are 127 and 85.

6 Let x L be the amount of 40% solution and y L be the amount of 15% solution. Equate the actual substance.

$$\begin{aligned} 0.4x + 0.15y &= 0.24 \times 700 \\ &= 168 \\ x + y &= 700 \end{aligned}$$

Multiply the second equation by 0.15.

$$0.4x + 0.15y = 168 \quad (1)$$
$$0.15x + 0.15y = 105 \quad (2)$$

$(1) - (2)$:

$$\begin{aligned} 0.25x &= 63 \\ x &= 63 \times 4 \\ &= 252 \\ 252 + y &= 700 \\ y &= 448 \end{aligned}$$

Use 252 L of 40% solution and 448 L of 15% solution.

7 Form two simultaneous equations.

$$x + y = 220 \quad (1)$$
$$x - \frac{x}{2} = y - 40$$
$$\frac{x}{2} - y = -40 \quad (2)$$

$(1) + (2)$:

$$\begin{aligned} \frac{3x}{2} &= 180 \\ x &= 120 \\ 120 + y &= 220 \\ y &= 100 \end{aligned}$$

They started with 120 and 100 marbles and ended with 60 each.

8 Let $\$x$ be the amount initially invested at 10% and y the amount initially invested at 7%. This earns $\$31\ 000$.

$$0.1x + 0.07y = 31\ 000$$

When the amounts are interchanged, she earns $\$1000$ more, i.e. $\$32\ 000$.

$$0.07x + 0.1y = 32\ 000$$

Multiply the first equation by 100 and the second equation by 70.

$$10x + 7y = 3\ 100\ 000 \quad (1)$$
$$4.9x + 7y = 2\ 240\ 000 \quad (2)$$

$(1) - (2)$:

$$\begin{aligned} 5.1x &= 860\ 000 \\ x &= \frac{860\ 000}{5.1} \approx 168\ 627.451 \end{aligned}$$

$$\begin{aligned} 10 \times 168\ 627.451 + 7y &= 3\ 100\ 000 \\ 1\ 686\ 274.51 + 7y &= 3\ 100\ 000 \\ 7y &= 1\ 413\ 725.49 \\ y &= 201\ 960.78 \end{aligned}$$

The total amount invested is

$$\begin{aligned} x + y &= 168\ 627.45 + 201\ 960.78 \\ &= \$370\ 588.23 \\ &= \$370\ 588 \end{aligned}$$

correct to the nearest dollar.

9 Let a be the number of adults and s the number of students who attended.

$$30a + 20s = 37\,000 \quad ①$$

$$a + s = 1600$$

$$20a + 20s = 1600 \times 20$$

$$= 32\,000 \quad ②$$

① − ②:

$$10a = 5000$$

$$a = 500$$

$$500 + s = 1600$$

$$s = 1100$$

500 adults and 1100 students attended the concert.